Towards Real-World Computationally Affordable Reformulation of Uncertainty-Aware Power System Optimisation

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Overview

Background—The global decarbonisation

The global decarbonisation (except the US) and the need for energy security are pushing the integration of renewable energy sources (RES) into the grid.
These RES are known to be highly uncertain:

- In 2012, National Renewable Energy Laboratory (NREL) [2] showed that, the load forecasting error had a normalised standard deviation of only 0.036, whereas the number for wind generation is 0.119, which is **3 times of that for load**.

For reliable grid operation, we need to incorperate uncertainty management into the grid. *However ...*

Background—Complexity of power system optimisation

Power system optimisation represents a collection of decision-making problems in the grid operation process.

- From the grid operators' perspective, two well-known examples are economic dispatch (ED) and unit commitment (UC).
- From the resource owners' perspective, this can be a strategic market bidding problem.

These problems are complex even in their deterministic forms, due to

1. The large problem scale, e.g., thousands of generators, nodes, and a more-than-30-step optimisation horizon. Not to say the millions+ grid-edge distributed energy resources (DERs);
2. Integer variables, such as those in the UC;
3. Nonlinearity and nonconvexity, such as the nonlinear battery power efficiency, or AC power flow equations.

Background—Complexity of power system optimisation

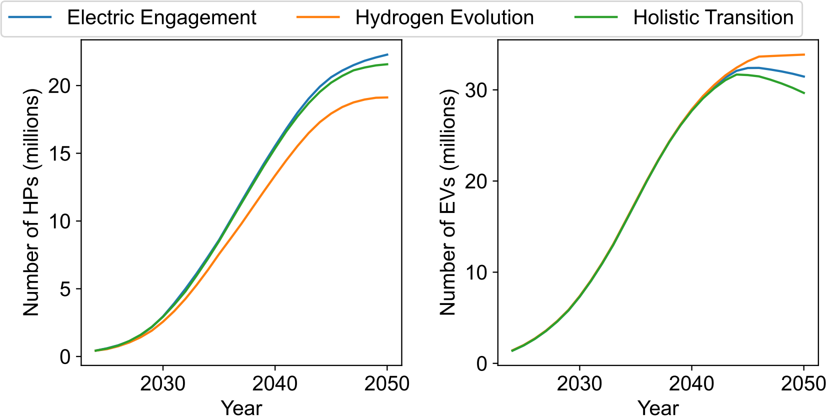
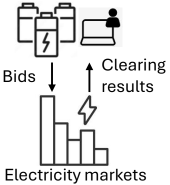


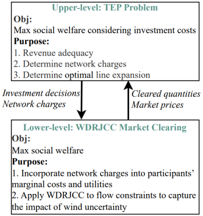
Figure: The estimated number of two representive DERs, namely electric vehicles (EVs) and heat pumps (HPs), in the three pathways that achieve the net-zero emission target in the UK [3].

The big picture: Computationally efficient uncertainty-aware methods

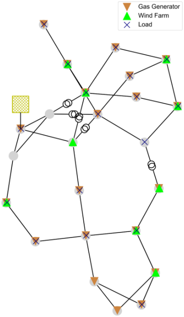
Storage strategic bidding



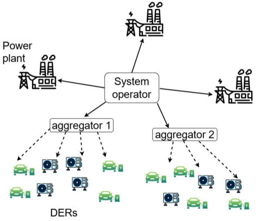
Transmission Expansion Planning (TEP)



Computationally efficient uncertainty-aware power system operation, e.g., economic dispatch (ED), unit commitment (UC), and optimal power flow (OPF)



Flexibility aggregation



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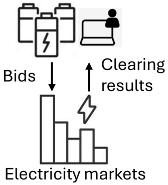
The Core: Accelerating uncertainty-aware grid dispatch

Storage strategic bidding



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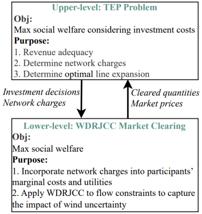

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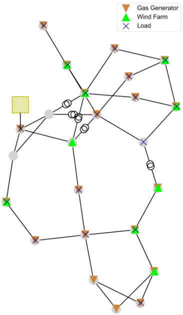

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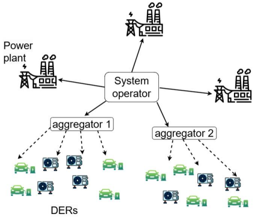

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Decision making under uncertainty

Many real-world decision-making problems involve uncertain parameters in their problem formulations.

1. The objective can involve uncertain parameters. For example, you are a **price-taker** storage operator and you want to minimise operation costs (maximise profits) under uncertain day-ahead market prices:

$$\min_{\boldsymbol{x} \in \mathcal{X}} \quad \mathbb{E}[c(\boldsymbol{x}, \boldsymbol{\xi})],$$

   where $\boldsymbol{x}$ is your decision variables s.t. some constraints in $\mathcal{X}$, $\boldsymbol{\xi}$ is the random vector (market prices here), and $c(\cdot)$ is some cost/profit function.

Decision making under uncertainty (cont.)

2. The constraints can also involve uncertain parameters. To deal with uncertainty in constraints, one can either
   2.1 Ensure constraint satisfaction for all possible scenarios, which is the robust optimisation (RO) approach (overly conservative...), or
   2.2 Ensure constraint satisfaction at a user-defined probability level, which is the chance constraint (CC) approach as below:

$$\begin{aligned} \min_{\boldsymbol{x}\in\mathcal{X}} \quad & c(\boldsymbol{x}) \\ \text{s.t.} \quad & \mathbb{P}[\boldsymbol{\xi} \notin \mathcal{S}(\boldsymbol{x})] \leq \epsilon \quad \text{A.K.A chance constraint (CC).} \end{aligned}$$

- $\mathbb{P}$ is a probability measure associated with $\boldsymbol{\xi}$, characterizing its distribution. $\mathcal{S}(\boldsymbol{x})$ is a set of constraints that couple $\boldsymbol{x}$ and $\boldsymbol{\xi}$, and ϵ is your desired risk level.
- When $\mathcal{S}(\boldsymbol{x})$ only has one constraint, the CC is individual CC (ICC). When $\mathcal{S}(\boldsymbol{x})$ contains several constraints, it becomes joint CC (JCC).
- **JCC is common and often desired due to the greater safety (also TODAY's main focus). Unfortunately JCC is often intractable and nonconvex.**

Wasserstein ambiguity

Another question naturally arises: *How can we get the distribution $\mathbb{P}$?*

- Collect historical data? This will require A LOT OF data to have a confident estimate of $\mathbb{P}$.
- And usually we only have an insufficient amount of data. In this case there is **ambiguity** towards the true $\mathbb{P}$!

A Wasserstein joint chance constraint (WJCC) can be expressed as:

$$\begin{aligned} \min_{\boldsymbol{x} \in \mathcal{X}} \quad & c(\boldsymbol{x}) \\ \text{s.t.} \quad & \sup_{\mathbb{P} \in \mathcal{F}_N(\theta)} \mathbb{P}[\boldsymbol{\xi} \notin \mathcal{S}(\boldsymbol{x})] \leq \epsilon, \end{aligned}$$

where $\mathcal{F}_N(\theta)$ is a collection of probability distributions that are within θ-Wasserstein distance to the historical data we have.

Right-hand-side uncertainty and left-hand-side uncertainty

WJCC can be divided into those with right-hand-side (RHS) uncertainty and left-hand-side (LHS) uncertainty.

1. When we say RHS, it means the safety set $\mathcal{S}(\boldsymbol{x})$ can be expressed as the following **separable** form:

$$\mathcal{S}(\boldsymbol{x}) := \left\{ \boldsymbol{\xi} \mid \boldsymbol{b}_p^\top \boldsymbol{\xi} + d_p - \boldsymbol{a}_p^\top \boldsymbol{x} \geq 0,\ p \in [P] \right\},$$

   - $\boldsymbol{b}_p$, d_p, and $\boldsymbol{a}_p$ are parameters. $\boldsymbol{\xi}$ is the random vector, and $\boldsymbol{x}$ is the decision vector.
   - There are P constraints indexed by $[P] = \{1, \cdots, P\}$ and thus a JCC.
   - The $\boldsymbol{x}$ and $\boldsymbol{\xi}$ appear in separate terms and thus RHS.

2. When we say LHS uncertainty, it means the decision variable $\boldsymbol{x}$ is multiplied with the random vector $\boldsymbol{\xi}$:

$$\mathcal{S}(\boldsymbol{x}) := \left\{ \boldsymbol{\xi} \mid (\boldsymbol{b}_p - \boldsymbol{A}_p^\top \boldsymbol{x})^\top \boldsymbol{\xi} + d_p - \boldsymbol{a}_p^\top \boldsymbol{x} \geq 0,\ p \in [P] \right\},$$

*Other terms in $\mathcal{S}(\boldsymbol{x})$ are known parameters.

Power system constraints with RHS uncertainty

When is this useful? to name a few:

1. Power system reserve constraints.

   The total up/downward reserve from all generators $g \in \mathcal{G}$ must cover the total imbalance $\mathbf{1}^\top \boldsymbol{e}$:

$$\sum_{g\in\mathcal{G}} \hat{r}_g \geq \mathbf{1}^\top \boldsymbol{e}, \quad \sum_{g\in\mathcal{G}} \check{r}_g \geq -\mathbf{1}^\top \boldsymbol{e}.$$

2. Line flow constraints w/o considering AGC re-dispatch, or line flow constraints under fixed AGC coefficients ($\boldsymbol{\alpha}$ **being parameters**):

$$\underline{f} \leq S^G(\boldsymbol{p}_g + \boldsymbol{\alpha}\mathbf{1}^\top \boldsymbol{e}) + S^W(\boldsymbol{\omega} + \boldsymbol{e}) \leq \overline{f},$$

   where S^G and S^W are the PTDF matrices in DCOPF. $\boldsymbol{\omega}$ is the RES forecast, and $\boldsymbol{e}$ is the uncertain forecasting error (imbalance).

3. AGC generator power limits: $\underline{p}_g \leq p_g + \alpha\mathbf{1}^\top \boldsymbol{e} \leq \overline{p}_g$.
4. Power availability of dispatchable RES: $p^{\text{RES}} \leq \overline{p}^{\text{RES}}$.

Challenges

The exact reformulation (if exists) of WJCC is non-convex, and the most efficient reformulation to-date is a mixed-integer programme (MIP). Although existing work has strengthened the formulation (such as the ExactS in [1]) to gain considerable speedup, the MIP nature still introduces issues such as

1. Bad worst-case performance, and
2. Lack of convexity (important for certain problems such as multi-level optimisation).

Convex approximation schemes exist (such as conditional value-at-risk, a.k.a. CVaR), but are still large-scale, leading to limited applicability to real-world industry-size applications.

Q: Can we have faster and convex reformulations for WJCC?

Proposed SFLA for RHS-WJCC

Motivated by ExactS [1] (which is a MIP), we propose a *Strengthened and Faster Linear Approximation (SFLA)* as the **fast and convex (in fact linear)** solution of RHS-WJCC, by exploiting valid inequalities!

Existing linear approximation (LA) or CVaR

$$s \geq 0, \boldsymbol{r} \geq \boldsymbol{0},$$

$$\epsilon N s - \sum_{i \in [N]} r_i \geq \theta N,$$

$$\kappa_i \left(\frac{\boldsymbol{b}_p^\top \boldsymbol{\xi}_i + d_p - \boldsymbol{a}_p^\top \boldsymbol{x}}{\|\boldsymbol{b}_p\|_*} \right) \geq s - r_i, \forall i \in [N], p \in [P].$$

Proposed SFLA

$$s \geq 0, \boldsymbol{r} \geq \boldsymbol{0},$$

$$\epsilon N s - \sum_{i \in [N]} r_i \geq \theta N,$$

$$\kappa_i \left(\frac{\boldsymbol{b}_p^\top \boldsymbol{\xi}_i + d_p - \boldsymbol{a}_p^\top \boldsymbol{x}}{\|\boldsymbol{b}_p\|_*} \right) \geq s - r_i, \forall i \in [N]_p, p \in [P],$$

$$\frac{q_p + d_p - \boldsymbol{a}_p^\top \boldsymbol{x}}{\|\boldsymbol{b}_p\|_*} \geq s, \forall p \in [P].$$

where we have $q_p :=$ the $(\lfloor \epsilon N \rfloor + 1)$-th smallest element of the set $\{\boldsymbol{b}_p^\top \boldsymbol{\xi}_i\}_{i \in [N]}$, and $[N]_p := \{i \in [N] \mid \boldsymbol{b}_p^\top \boldsymbol{\xi}_i < q_p\}$. This form is also known as quantile strengthening.

Proposed SFLA for RHS-WJCC

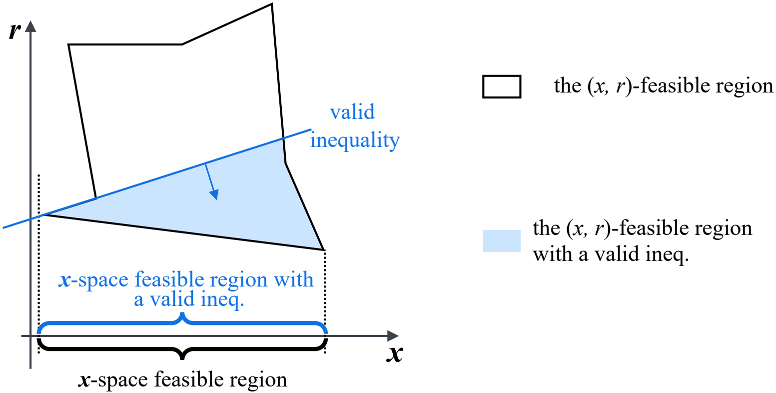
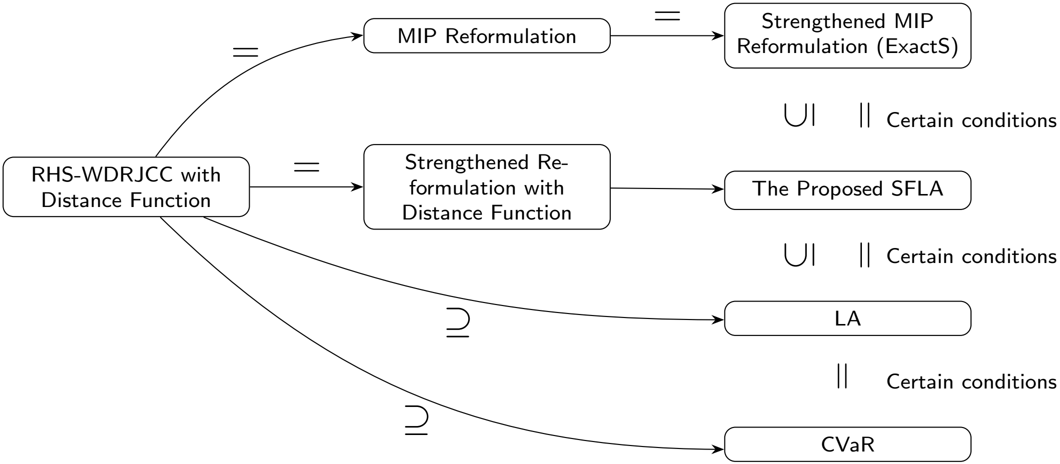
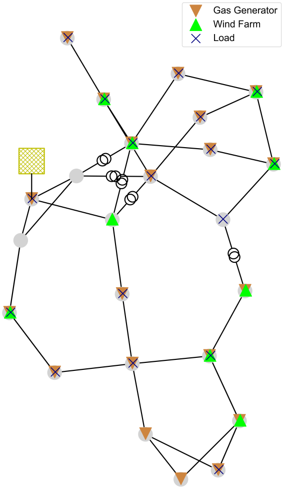


Figure: Tractable reformulation of WJCC usually requires introducing ancillary variables ($\boldsymbol{r}$) in addition to the original decision variable $\boldsymbol{x}$. Sorely removing the ancillary feasible region is **valid** because the objective $c(\boldsymbol{x})$ is independent of $\boldsymbol{r}$. The reduced search space is the key to speedup.

Proposed SFLA for RHS-WJCC (comparisons with other methods)

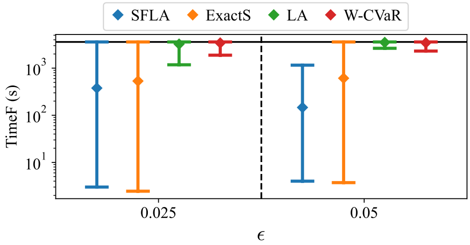


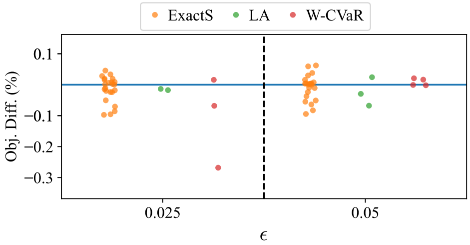
Applying SFLA to unit commitment on IEEE 24-bus network



- Objective: minimising the total generation quadratic fuel cost+start-up/shut-down cost.
- Deterministic constraints: generator operational constraints.
- RHS-WJCC: 1) Total system reserve and 2) line flow constraints.
- 100 generators and an 28-step optimisation horizon.
- 30 random runs per problem parameter setting, increasing to 150 when necessary.

Applying SFLA to unit commitment on IEEE 24-bus network





The proposed SFLA is faster than benchmarks and has comparable optimality. 3.8× faster compared to ExactS (averaged through many sensitivity runs with each repeating 150 times), 20× faster than LA and CVaR.

Challenges re-highlighted for LHS-WJCC

Recall the LHS-WJCC:

$$\mathcal{S}(\boldsymbol{x}) := \left\{ \boldsymbol{\xi} \mid (\boldsymbol{b}_p - \boldsymbol{A}_p^\top \boldsymbol{x})^\top \boldsymbol{\xi} + d_p - \boldsymbol{a}_p^\top \boldsymbol{x} \geq 0,\ p \in [P] \right\}.$$

LHS-WJCC is in general hard to solve. Exact reformulation only exists for **homogeneous constraint coefficients,** i.e., $\boldsymbol{b}_p = \boldsymbol{b}$ and $\boldsymbol{A}_p = \boldsymbol{A}$ for all $p \in [P]$, or need to introduce **artifitial degeneracy** that may lead to extra conservativeness.

Much more difficult to solve compared to RHS-WJCC!

Power system constraints with LHS uncertainty

But LHS-WJCC is common in power system dispatch. Specifically, it is desired to to co-optimise the AGC factors, i.e., α **being decision variables**:

1. line flow constraints:

$$\underline{f} \leq S^G(\boldsymbol{p}_g + \boldsymbol{\alpha}\mathbf{1}^\top \boldsymbol{e}) + S^W(\boldsymbol{\omega} + \boldsymbol{e}) \leq \overline{f},$$

   where S^G and S^W are the PTDF matrices in DCOPF. $\boldsymbol{\omega}$ is the RES forecast, and $\boldsymbol{e}$ is the uncertain forecasting error (imbalance).

2. AGC generator power limits: $\underline{p}_g \leq p_g + \alpha_g \mathbf{1}^\top \boldsymbol{e} \leq \overline{p}_g$.

Proposed FICA for LHS-WJCC

- We propose a *Faster Inner Convex Approximation (FICA)* method as the **fast and convex** (conic or linear) solution to LHS-WJCC.
- The core idea is still valid inequalities (quantile strengthening), but we exploit the **one-dimensional** structure of the power system dispatch problems, such as in

$$\underline{p}_g \leq p_g + \alpha_g \mathbf{1}^\top \boldsymbol{e} \leq \overline{p}_g.$$

because there is *only one* multiplication between a one-dimensional decision variable α_g and a one-dimensional random variable $\mathbf{1}^\top \boldsymbol{e}$. This property enjoys the following theoretical property:

Proposition 1. Suppose $e_1 \leq \ldots \leq e_N$, then the j-th smallest element of $\{\alpha_g e_i\}_{i\in[N]}$ is equal to $\min\{\alpha e_j,\ \alpha e_{N-j+1}\}$ for any $\alpha_g \in \mathbb{R}$.

which is important to derive the q_p and $[N]_p$ for the valid inequalities

Proposed FICA for LHS-WJCC

We show that, as long as a LHS-WJCC contains **some** one-dimensional structure LHS, then the solution process of the whole LHS-WJCC can be significantly accelerated.

CVaR

$$
\begin{aligned}
& s \geq 0, \boldsymbol{r} \geq \boldsymbol{0}, \\
& \epsilon N s - \sum_{i \in [N]} r_i \geq \theta N \max_{p \in [P]} \|\boldsymbol{b}_p - \boldsymbol{A}_p^\top \boldsymbol{x}\|_*, \\
& (\boldsymbol{b}_p - \boldsymbol{A}_p \boldsymbol{x})^\top \boldsymbol{\xi}_i + d_p - \boldsymbol{a}_p^\top \boldsymbol{x} \geq s - r_i, \\
& \qquad \forall i \in [N], p \in [P].
\end{aligned}
$$

Proposed FICA

$$
\begin{aligned}
& s \geq 0, \boldsymbol{r} \geq \boldsymbol{0}, \\
& \epsilon N s - \sum_{i \in [N]} r_i \geq \theta N \max_{p \in [P]} \|\boldsymbol{b}_p - \boldsymbol{A}_p^\top \boldsymbol{x}\|_*, \\
& (\boldsymbol{b}_p - \boldsymbol{A}_p^\top \boldsymbol{x})^\top \boldsymbol{\xi}_i + d_p - \boldsymbol{a}_p^\top \boldsymbol{x} \geq s - r_i, \\
& \qquad \forall i \in [N]_p, \ p \in [P]^*, \\
& (\boldsymbol{b}_p - \boldsymbol{A}_p^\top \boldsymbol{x})^\top \boldsymbol{\xi}_i + d_p - \boldsymbol{a}_p^\top \boldsymbol{x} \geq s - r_i, \\
& \qquad \forall i \in [N], \ p \in [P] \setminus [P]^*, \\
& q_p(\boldsymbol{x}) + d_p - \boldsymbol{a}_p^\top \boldsymbol{x} \geq s, \qquad \forall p \in [P]^*.
\end{aligned}
$$

where we let $[P]^*$ collect all one-dimensional LHS (or RHS) constraints.

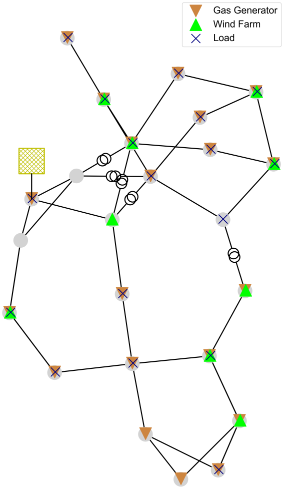
Theorem. CVaR and FICA always have the same optimal value for any objevtive $c(\boldsymbol{x})$.

Note that, unlike RHS-WJCC where we have ExactS, there is no reformulation that is faster than CVaR under the same approximation quality.

Applying FICA to economic dispatch (with AGC) on IEEE-24 network

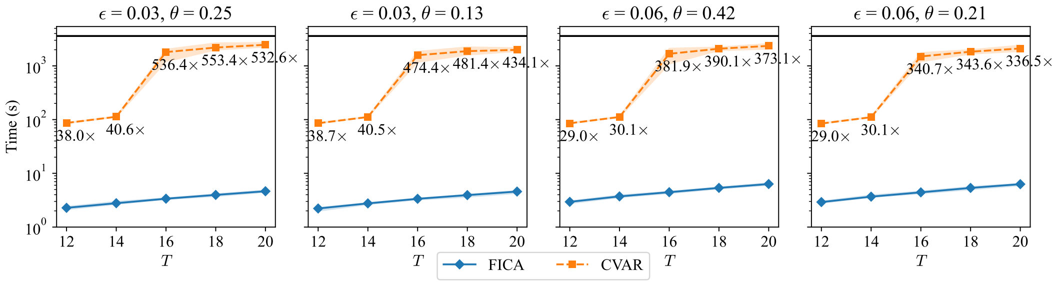
$$
\begin{aligned}
\min_{\boldsymbol{p},\boldsymbol{\alpha}} \quad & \mathcal{C}(\boldsymbol{p},\boldsymbol{\alpha}) && && \text{(1a)}\\
\text{s.t.} \quad & \boldsymbol{p},\ \boldsymbol{\alpha} \in \mathcal{X}, && && \text{(1b)}\\
& \mathbf{1}^\top(\boldsymbol{p}_t - \boldsymbol{d}_t) = 0, && \forall t \in [T], && \text{(1c)}\\
& \mathbf{1}^\top \boldsymbol{\alpha}_t = 1, && \forall t \in [T], && \text{(1d)}\\
& -\mathbf{1} \le \boldsymbol{\alpha}_t \le \mathbf{1}, && \forall t \in [T], && \text{(1e)}\\
& \underline{\boldsymbol{p}}_t \le \boldsymbol{p}_t \le \overline{\boldsymbol{p}}_t, && \forall t \in [T], && \text{(1f)}\\
\inf_{\mathbb{P}\in\mathcal{F}(\theta)} \mathbb{P} & \left\{ \begin{aligned} & \underline{\boldsymbol{p}}_t \le \boldsymbol{p}_t - \sum_{w\in\mathcal{W}} e_{t,w}\boldsymbol{\alpha}_t \le \overline{\boldsymbol{p}}_t, \quad \forall t \in [T], \\ & \underline{\boldsymbol{f}} \le \boldsymbol{S}^G \tilde{\boldsymbol{p}}_t + \boldsymbol{S}^W(\boldsymbol{\omega}_t + \boldsymbol{e}_t) - \boldsymbol{S}^D \boldsymbol{d}_t \le \overline{\boldsymbol{f}}, \forall t \in [T] \end{aligned} \right\} \ge 1-\epsilon, && && && \text{(1g)}
\end{aligned}
$$

Applying FICA to economic dispatch (with AGC) on IEEE-24 network



- 38 generators and 10 wind farms.
- 10 random runs per problem parameter setting.

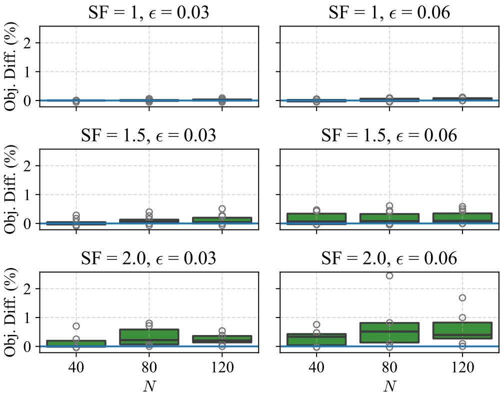
Applying FICA to economic dispatch (with AGC) on IEEE-24 network



- When the optimisation horizon T exceeds 16, FICA achieves 500× speedup compared to CVaR in solving the LHS-WJCC economic dispatch problem.
- Besides, we have observed FICA being super memory-saving, due to the $[N]_p$ set that reduces the N constraints in $[N]$ to $\lfloor \epsilon N \rfloor$.

* The proportion of the one-dimensional constraints is only 50%.

Applying FICA to economic dispatch (with AGC) on IEEE-24 network



We have verified that **FICA achieves the same optimality as CVaR for all experiments**.
So here we **specifically** compare FICA (as a convex inner-approximation) to the exact reformulation method with $T = 1$ (the exact method is hard to solve).

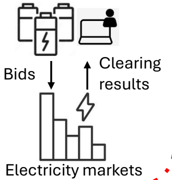
- The optimality gap is below 1% for most cases.

Summary of takeaways

1. We exploit valid inequalities to derive faster convex inner-approximation to WJCC, without compromising optimality compared to well-known approximation such as CVaR (rigorously proved).
2. The speedup can be up to 500$\times$ (in FICA, there is no existing work that strengthen CVaR in a LHS-WJCC).
3. As long as a WJCC contains some special-structure constraints, such as RHS or one-dimensional, then SFLA and FICA can be applied.

Extension 1: Lower-level JCC in multi-level strategic bidding

Storage strategic bidding



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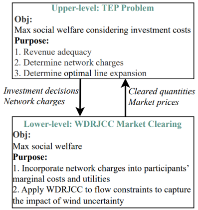
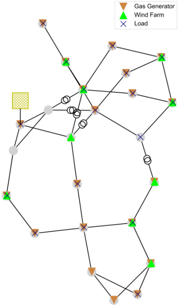
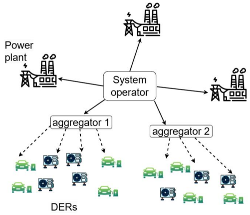


Computationally efficient uncertainty-aware power system operation, e.g., economic dispatch (ED), unit commitment (UC), and optimal power flow (OPF)



Flexibility aggregation



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Extension 1: Lower-level JCC in multi-level strategic bidding

The electricity market can also clear with the RHS-WJCC as the previous cases.

When a **price-maker** storage operator wants to participate this market, it becomes a bilevel optimisation problem:

- In the upper level, the storage operator determine the bids $\boldsymbol{x}$, subject to some storage operation constraints $\mathcal{X}$. The objective is to maximise the profit $F(\boldsymbol{x}, \boldsymbol{y})$, which depends on 1) the bid $\boldsymbol{x}$ and 2) the market clearing result $\boldsymbol{y}$.
- In the lower-level, the market clears by minimising the system-wide cost $f(\boldsymbol{x}, \boldsymbol{y})$, subject to RHS-WJCC.

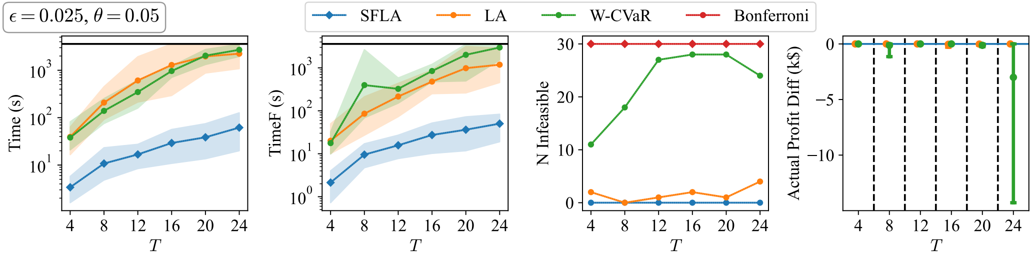
Extension 1: Lower-level JCC in multi-level strategic bidding

Mathematically, this problem can be formulated as:

$$
\begin{aligned}
\max_{\mathbf{x}\in\mathcal{X}} \quad & F(\boldsymbol{x}, \boldsymbol{y}) \\
\text{s.t.} \quad & \boldsymbol{y} \in \arg\min_{\boldsymbol{y}} f(\boldsymbol{x}, \boldsymbol{y}) \\
& \text{s.t.} \\
& \mathbb{P}_{\boldsymbol{\xi}}\left(\boldsymbol{\xi} \notin \mathcal{S}(\boldsymbol{x}, \boldsymbol{y})\right) \geq 1 - \epsilon \\
& h(\boldsymbol{x}, \boldsymbol{y}) \leq 0
\end{aligned}
$$

- Common solutions exploit the KKT conditions to reformulate the bilevel problem into a single-level equivalence, which requires a convex lower-level problem.
- Convex approximations, such as LA, CVaR, and the proposed SFLA and FICA become important.

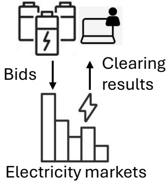
Extension 1: Case studies



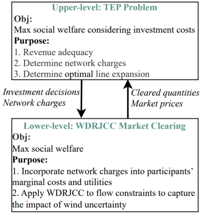
- The proposed SFLA achieves a speedup of up to 100× in solving to optimality (*Time*) compared to LA and CVaR,
- The proposed SFLA always achieve no-less profits than benchmarks.

Extension 2: Two-stage JCC in flexibility aggregation

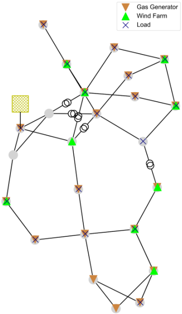
Storage strategic bidding



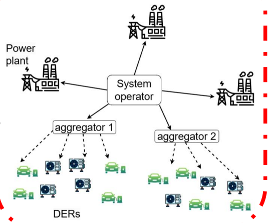
Transmission Expansion Planning (TEP)



Computationally efficient uncertainty-aware power system operation, e.g., economic dispatch (ED), unit commitment (UC), and optimal power flow (OPF)



Flexibility aggregation



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Extension 2: Aggregation

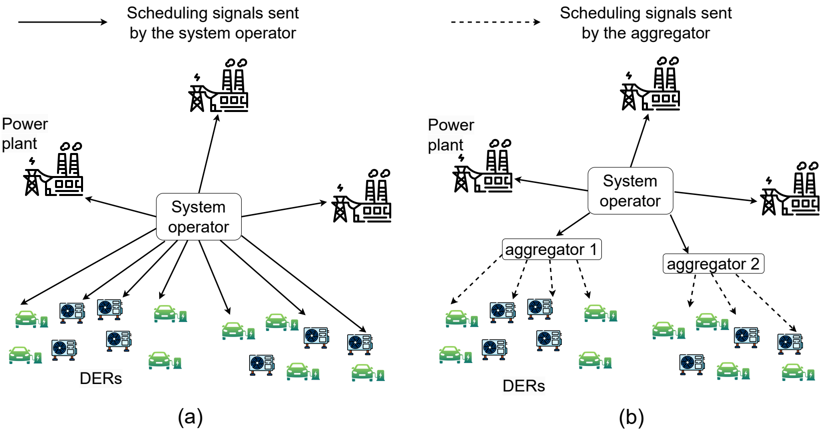


Figure: Illustration of the aggregation process. (a) Direct system scheduling. (b) Hierarchical scheduling through aggregators.

Extension 2: Aggregation

Say there is a set of DERs connected to a distribution network. The decision variables of these DERs are denoted by $\boldsymbol{x}$, which are subject to their own (temporal) operational constraints and are spatially coupled through network constraints:

$$f_1(\boldsymbol{x}) \leq 0, \ldots, f_M(\boldsymbol{x}) \leq 0. \quad M \text{ tends to be a large number.}$$

Aggregation wants to characterise the aggregated DER power p^{Agg}, usually the point of common coupling (PCC) with the upstream network. Through power flow equations, we have

$$p^{\mathrm{Agg}} = h(\boldsymbol{x})$$

The objective of aggregation is thus to characterise

$$\Omega^{\mathrm{Agg}} := \left\{ p^{\mathrm{Agg}} \mid p^{\mathrm{Agg}} = h(\boldsymbol{x}), f_1(\boldsymbol{x}) \leq 0, \ldots, f_M(\boldsymbol{x}) \leq 0. \right\}$$

Extension 2: Aggregation under uncertainty

Due to the existence of uncertainty, the problem becomes:

$$\Omega^{\text{Agg}} := \left\{ p^{\text{Agg}} \mid p^{\text{Agg}} = h(\boldsymbol{x}^d, \boldsymbol{\xi}), f_1(\boldsymbol{x}^d, \boldsymbol{\xi}) \leq 0, \ldots, f_M(\boldsymbol{x}^d, \boldsymbol{\xi}) \leq 0. \right\}$$

From the perspective of the upstream network, the focused reliability is on **the aggregated power**.

Then, similar to the commonly used two-stage stochastic programming or two-stage RO in UC or ED, we can also transfer this to DER aggregation as an **approximated modelling** to the real aggregation-disaggregation process:

$$\Omega^{\text{Agg}}(\epsilon) := \left\{ p^{\text{Agg}} \mid \mathbb{P}\left\{ \exists \boldsymbol{x}^d, \text{s.t.}\, p^{\text{Agg}} = h(\boldsymbol{x}^d, \boldsymbol{\xi}), f_1(\boldsymbol{x}^d, \boldsymbol{\xi}) \leq 0, \ldots, f_M(\boldsymbol{x}^d, \boldsymbol{\xi}) \leq 0 \right\} \geq 1 - \epsilon \right\},$$

where $\boldsymbol{x}^d$ represents the disaggregation action (the power dispatch for individual DERs).

Extension 2: Proposed method

- If we aply standard two-stage (even approximation) method, then the problem will be challenging because there are too many constraints in the second stage.
- However, when there is no uncertainty, existing work has demonstrated the effective approximation models such as virtual battery (VB) to characterise Ω^{Agg}:

$$\exists \Omega^{\text{Approx}} := \{\boldsymbol{p}^{\text{Agg}} \mid \boldsymbol{A}\boldsymbol{p}^{\text{Agg}} \leq \boldsymbol{b}^{\text{Agg}}\} \text{ with } \boldsymbol{A} \in \mathbb{R}^{D \times T} \text{ and } \boldsymbol{b} \in \mathbb{R}^{D},$$
$$\text{such that } D \ll M \text{ and } \Omega^{\text{Approx}} \subseteq \Omega^{\text{Agg}}.$$

Specifically, one can fix the slope $\boldsymbol{A}$ and tune the intercept $\boldsymbol{b}$ to reach reasonable approximation quality, such as in VB or virtual generator (VG) models.

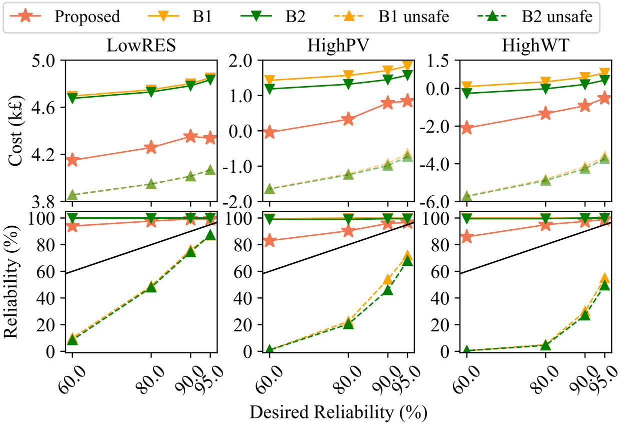
Extension 2: Proposed method (Cont)

- This observation motivates us to start from Ω^{Approx}, instead of the original Ω^{Agg}
- We can arrive at the following result:
  Theorem. If $\Omega^{\text{Approx}} \subseteq \Omega^{\text{Agg}}$, then

$$\Omega^{\text{Approx}}(\epsilon) := \{\boldsymbol{p}^{\text{Agg}} \mid \mathbb{P}(\boldsymbol{A}\boldsymbol{p}^{\text{Agg}} \leq \boldsymbol{b}) \geq 1 - \epsilon\} \subseteq \Omega^{\text{Agg}}(\epsilon)$$

  If $\boldsymbol{A}$ is chosen as a priori, we will only need to deal with **a single-level small-scale RHS-JCC**!

Extension 2: Case studies



- Compare against single-level benchmarks in existing studies:
  approximating the original two-stage JCC as a single-stage JCC, and solved through 1) Bonferroni approximations (B1 and B2) and 2) ICC but with the same risk level as JCC (less reliable, B1unsafe and B2unsafe)
- Achieved a better balance between reliability and optimality.

Conclusions

- The increasing uncertainty in the grid, the growing problem scale, and the need from other applications (such as bidding), are calling for **computationally efficient** and (preferably convex) **uncertainty-aware** grid dispatch algorithms.
- Valid inequalities are an extremely powerful tool to speed up the solution process of WJCC (can be $500\times$ faster).
- These inequalities can be derived by either
  1. Exploiting (even partially) the special structure of the problem, e.g., the RHS or one-dimensional structure in SFLA and FICA;
  2. Approximating the original problem with a simpler one that has the special structure, such as our aggregation problems.

Find more information in our paper:

1. SFLA: Yihong Zhou, Yuxin Xia, Hanbin Yang, and Thomas Morstyn. “Strengthened and Faster Linear Approximation to Joint Chance Constraints with Wasserstein Ambiguity.” Available at arXiv: `https://arxiv.org/abs/2412.12992` (codes and data available).
2. FICA: Yihong Zhou, Hanbin Yang, and Thomas Morstyn. “FICA: Faster Inner Convex Approximation of Chance Constrained Grid Dispatch with Decision-Coupled Uncertainty.” Being processed by arXiv. (codes and data will be available).
3. Flexibility aggregation: Yihong Zhou, Chaimaa Essayeh, and Thomas Morstyn. “Aggregated Feasible Active Power Region for Distributed Energy Resources with a Distributionally Robust Joint Probablistic Guarantee.” IEEE Transactions on Power Systems, 2024.

Questions?

Experiences

- Try to decompose problems.
- Try to express more.
- Try to challenge more (even other domains, as long as politely).
- Think more.

References

[1] Nam Ho-Nguyen, Fatma Kılınç-Karzan, Simge Küçükyavuz, and Dabeen Lee. Distributionally robust chance-constrained programs with right-hand side uncertainty under wasserstein ambiguity. *Mathematical Programming*, pages 1–32, 2022.

[2] Bri-Mathias Hodge, Anthony Florita, Kirsten Orwig, Debra Lew, and Michael Milligan. Comparison of wind power and load forecasting error distributions. Technical report, National Renewable Energy Lab.(NREL), Golden, CO (United States), 2012.

[3] National Grid ESO. Uk future energy scenarios. https://www.nationalgrideso.com/future-energy/future-energy-scenarios.

Thank you!